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Robust Tracking Control Using Fuzzy Disturbance Observer for Wheeled Mobile Robots with Skidding and Slipping

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Abstract This paper proposes a robust tracking controller based on the Fuzzy Disturbance Observer (FDO) for a Wheeled Mobile Robot (WMR) with unknown skidding and slipping. The proposed method provides disturbance-free techniques for stability analysis. In our previous work [1], we proposed an extended state-observer approach to robust tracking control for wheeled mobile robots with skidding and slipping. Even though satisfying performances were shown and the proposed method was verified in [1], the derivatives of disturbance should go to zero as time passes in order to guarantee performance. This is a very critical assumption. The method proposed in this paper overcomes this problem using universal approximation with a fuzzy model. Thus, the condition that disturbance should disappear with time is not required anymore. Furthermore, the proposed method can be used more widely than that shown in the previous work. This is guaranteed by a Lyapunov-theory-based stability analysis, and performance is verified by simulation results.

Keywords Wheeled Mobile Robot, Skidding, Slipping, Robust Tracking Control, Fuzzy Disturbance Observer

1. Introduction

Stabilization of nonholonomic WMRs has attracted much attention due to the inherent nonlinearity in the dynamics of these robots. In particular, studies of WMR with skidding and slipping have been carried out for tracking control, since skidding and slipping can happen in various real environments, such as on wet roads and icy roads and in rapid cornering, and the nonholonomic constraints can be perturbed [1-7]. In order to achieve the tracking control, not only kinematics but also dynamics of WMRs should be defined. Hence, the kinematics and dynamics of WMRs with skidding and slipping have been redefined; skidding and slipping are defined by input-additive and unmatched perturbations of the kinematics and dynamics of a WMR [3].

Based on this redefinition, the objective of previous studies [2, 5, 6] was to improve the robustness against disturbance in tracking control using robust techniques. In order to prove the stability of system, previous studies have needed to choose proper norm bounds. Since the dynamics of a nonholonomic WMR show highly nonlinear behaviour, it is difficult to estimate the proper norm bound.

The use of a disturbance observer can resolve these difficulties. Disturbance estimation techniques such as Extended State Observer (ESO) [13-15] or Nonlinear Disturbance Observer (NDO) [16, 17] are particularly crucial for disturbance attenuation. Disturbance-observer-based control has the advantage of compensating many uncertainties in control systems, which are unmeasurable. The present author has earlier proposed an extended state-observer approach to robust tracking control for wheeled mobile robots with skidding and slipping [1].

However, from a control theoretical viewpoint, the previous work still presents a problem: it relies on the assumption that the derivative of disturbances should converge to zero with time in order to guarantee the stability of the control system [1, 13-17]. This means that disturbances disappear with time. In practice, nevertheless, real skidding and slipping do not disappear with time, which to say that the derivatives of disturbances never go to zero with time. Without the assumption, the controller proposed in the previous work not only cannot achieve the desired goal and but will even make WMRs unstable. This is a fatal weakness for implementation.

In order to overcome the problem, the robust tracking controller based on the FDO method for a WMR with skidding and slipping, which occur throughout the operation of WMRs, is proposed as an expansion of our previous work in this paper. With universal approximation of fuzzy modelling, FDO is adopted to design a disturbance observer. Although disturbances represent highly nonlinear behaviour, the proposed method approximates the disturbances. In addition, it can analyse the stability and design a controller free from the assumption of disturbances. The performance of the proposed method is verified by a comparison with the previous work.

This paper is organized as follows. Section 2 shows the modified model of a nonholonomic WMR with skidding and slipping and the FDO for a general nonlinear system. A design and stability analysis of the proposed FDO-based tracking control details is described in section 3. Simulation results of the proposed controller with a WMR are given in section 4, followed by the conclusion in section 5.

2. Fuzzy disturbance observer design for the WMR with skidding and slipping

2.1 Description of the modified WMR with skidding and slipping

We consider the perturbed nonholonomic constraints with skidding and slipping as follows:

$$\begin{aligned} &\dot{y}\cos\phi - \dot{x}\sin\phi - d\dot{\phi} = \mu \\ &\dot{x}\cos\phi + \dot{y}\sin\phi + b\dot{\phi} = r(\dot{\theta}_r - \zeta_r) \\ &\dot{x}\cos\phi + \dot{y}\sin\phi - b\dot{\phi} = r(\dot{\theta}_l - \zeta_l) \end{aligned} \tag{1}$$

where μ indicates the lateral skidding velocity in mobile robots, which can easily be caused by the centrifugal force generated when mobile robots travel at high speed on a wet or icy road with corners. $\zeta = [\zeta_r \quad \zeta_l]$ denote perturbed angular velocities due to slipping of the two actuated wheels, respectively.

The kinematics of mobile robots considering skidding and slipping is obtained as in [1]:

$$\dot{q} = S(q)(z - \xi) + \varphi(q,\mu) \tag{2}$$

where $q = [x \quad y \quad \phi \quad \theta_r \quad \theta_l]^T$; The three variables x, y, ϕ describe the position and orientation of the platform, and the two variables θ_r, θ_l specify the angular positions for the driving wheels, $\xi = [\xi_v \quad \xi_\omega]^T$; $\xi_v = r(\zeta_r + \zeta_l)/2$ is the longitudinal slip velocity and $\xi_\omega = r(\zeta_r - \zeta_l)/(2b)$ is the yaw rate perturbation due to slippage of the wheels, $S(q)$ is the matrix to represent internal states z about the kinematics of wheeled mobile robots, and $\varphi(q,\mu) = [-\mu\sin\phi \quad \mu\cos\phi \quad 0 \quad \zeta_r \quad \zeta_l]^T$ denotes the mismatched disturbances vector induced from the perturbed nonholonomic constraints. Then, the disturbances satisfy assumption 1 to be controllable in general.

Assumption 1 [5]: The perturbations ξ_v, ξ_ω and $\varphi(q,\mu)$ are bounded as $\|\xi_v\| < \beta_1$, $\|\xi_\omega\| < \beta_2$ and $\|\varphi(q,\mu)\| = |\mu| < \beta_3$ because of $\left\|[-\sin\phi \quad \cos\phi]^T\right\| = 1$ where $\beta_i \ (i = 1,2,3)$ are unknown positive constants, and their first derivatives are also bounded as $\|\dot{\xi}_v\| < \beta_{d1}$, $\|\dot{\xi}_\omega\| < \beta_{d2}$ and $\|\dot{\varphi}(q,\mu)\| = |\dot{\mu}| < \beta_{d3} + \beta_3(|\omega| + \beta_2)$ with $|\dot{\mu}| < \beta_{d3}$ where $\beta_{di} \ (i = 1,2,3)$ are unknown positive constants.

The dynamics of mobile robots considering skidding and slipping is obtained as in [1]:

$$\begin{aligned} &H(q)(\dot{z} - \dot{\xi}) + F_1(q,\dot{q})(z - \xi) + F_2(q)\dot{\varphi}(q,\mu) \\ &+F_3(q)\varphi(q,\mu) + F_4(q) + \tau_d = \tau \end{aligned} \tag{3}$$

where $H(q) = (S^T(q)E(q))^{-1}S^T(q)M(q)S(q)$, $F_1(q,\dot{q}) = (S^T(q)E(q))^{-1}S^T(q)(M(q)\dot{S}(q) + V(q)S(q))$, $F_2(q) = (S^T(q)E(q))^{-1}S^T(q)M(q)$, $F_3(q) = (S^T(q)E(q))^{-1}S^T(q)V(q)$, $F_4(q) = (S^T(q)E(q))^{-1}S^T(q)G(q)$, $\tau = \begin{bmatrix}\tau_r & \tau_l\end{bmatrix}^T$ are the situated torque as the control input, $\tau_d \in \Re^l$ is the disturbance of an input vector, $M(q) \in \Re^{n\times n}$ is a symmetric and positive definite inertia matrix, $V(q) \in \Re^{n\times n}$ is the centripetal and Coriolis matrix, $G(q) \in \Re^n$ is the gravitational vector, $E(q) \in \Re^{n\times l}$ is an input transformation matrix, and $l = n - m$ ..

We can rewrite (3) as follows:

$$\begin{aligned}\dot{z} &= A(q,\dot{q})z + B(q)\tau - A(q,\dot{q})\xi + B(q)\dot{\xi} \\ &\quad - H^{-1}(q)F_2(q)\dot{\varphi}(q,\mu) - H^{-1}(q)F_3(q)\varphi(q,\mu) \\ &\quad - H^{-1}(q)F_4(q) + H^{-1}(q)\tau_d\end{aligned} \tag{4}$$

where $A(q,\dot{q}) = -H^{-1}(q)F_1(q,\dot{q})$, $B(q) = H^{-1}(q)$.

Since $A(q,\dot{q})$, $B(q)$ is changed by q, which is affected by unknown skidding and slipping, we should consider parameter uncertainties and parameter variation of the dynamics of the WMR. Therefore, (4) is rewritten as follows:

$$\begin{aligned}\dot{z} &= A(q,\dot{q})z + B(q)\tau - A(q,\dot{q})\xi + B(q)\dot{\xi} \\ &\quad - H^{-1}(q)F_2(q)\dot{\varphi}(q,\mu) - H^{-1}(q)F_3(q)\varphi(q,\mu) \\ &\quad - H^{-1}(q)F_4(q) + H^{-1}(q)\tau_d + \Delta A(q,\dot{q})z + \Delta B(q)\tau\end{aligned} \tag{5}$$

where $\Delta A(q,\dot{q})$, $\Delta B(q)$ are parameter uncertainties and parameter variation by skidding and slipping of the WMR.

Definition 1: Parameter uncertainties, parameter variation, input disturbance, and skidding and slipping of the mobile robot are regarded as disturbances as follows:

$$\begin{aligned}D(z,\tau) &= -A(q,\dot{q})\xi + B(q)\dot{\xi} - H^{-1}(q)F_2(q)\dot{\varphi}(q,\mu) \\ &\quad - H^{-1}(q)F_3\varphi(q,\mu) + H^{-1}(q)\tau_d \\ &\quad - H^{-1}(q)F_4(q) + \Delta A(q,\dot{q})z + \Delta B(q)\tau\end{aligned} \tag{6}$$

Using definition 1, we can rewrite (5) simply as

$$\dot{z} = A(q,\dot{q})z + B(q)\tau + D(z,\tau) \tag{7}$$

where $D(z,\tau)$ is the total perturbation. Rewriting (5) as (7) gives the advantage of easy application of robust control and observer-based control. The control can thus cancel the total perturbation exactly and fast. But this method is difficult exactly because disturbance is unknown and unpredictable; so, we need to approximate the disturbance using an FDO scheme without assumption 1.

2.2 Fuzzy disturbance observer

We now describe a fuzzy system to approximate the disturbance. The basic configuration of a fuzzy logic system consists of a fuzzifier, some fuzzy IF-THEN rules, a fuzzy inference engine and a defuzzifier. The fuzzy inference engine uses the fuzzy IF-THEN rules to perform a mapping from an input linguistic vector $x = (x_1, x_2, \ldots, x_n)^T \in R^n$ to an output variable $y \in R$. The i th fuzzy rule is written as

$$R^{(j)}: IF\ x_1\ is\ A_1^j and \cdots and\ x_n\ is\ A_n^j THEN\ y\ is\ y^j \tag{8}$$

where $A_1^j, A_2^j, \ldots, A_n^j$ are fuzzy variables and y^i is a singleton number. By using a strategy of singleton fuzzification, product inference and centre-of-gravity defuzzification, the fuzzy system becomes

$$y(x) = \frac{\sum_{j=1}^m y^j\left(\prod_{i=1}^n \mu_{A_i^j}(x_i)\right)}{\sum_{j=1}^m\left(\prod_{i=1}^n \mu_{A_i^j}(x_i)\right)} \tag{9}$$

where $\mu_{A_i^j}(x_i)$ is the membership function of the linguistic variable x_i and m is the number of fuzzy rules. By introducing the concept of fuzzy logic systems in [8], (9) can be rewritten as

$$y(x) = \theta^T\xi(x) \tag{10}$$

where $\theta = \left(h^1, \cdots, h^m\right)^T$ are adjustable parameter vectors, $\xi(x) = \left(\xi^1(x), \cdots, \xi^m(x)\right)^T$;

$$\xi(x) = \frac{\prod_{i=1}^n \mu_{A_i^j}(x_i)}{\sum_{j=1}^m\left(\prod_{i=1}^n \mu_{A_i^j}(x_i)\right)}, \quad j = 1, 2, \cdots, m \tag{11}$$

are fuzzy basis functions (FBFs).

The task of this section is to develop an FDO and a tuning method so that the developed disturbance observer is guaranteed to monitor and accurately represent the disturbance that occurs [8]. To proceed with the development, the following assumption is required.

Assumption 2 [8]: Let x belong to a compact set M_x. The optimal parameter vector θ^* is defined as

$$\theta^* = \arg\min_{\hat{\theta}\in M_\theta}\left[\sup_{x\in M_x}\left|D(z,\tau) - \hat{D}(z,\tau\mid\hat{\theta})\right|\right] \tag{12}$$

and it is assumed that the optimal parameter vector θ^* lies in a convex region:

$$M_\theta = \{\theta \mid \|\theta\| \le m_\theta\} \tag{13}$$

where m_θ is a design parameter.

For system (7), the fuzzy disturbance observer is designed as follows:

$$\dot{\mu} = -\sigma\mu + \sigma z + A(q,\dot{q})z + B(q)\tau + \hat{D}(z,\tau \mid \hat{\theta}) \tag{14}$$

where μ is an observer state, a disturbance observation error $\zeta \equiv z - \mu$ and $\hat{\Omega}(x,u \mid \hat{\theta})$ is the FDO and σ is a positive constant. To construct $\hat{D}(z,\tau \mid \hat{\theta})$, which is guaranteed to monitor an unknown $D(z,\tau)$, a suitable tuning method must be provided for the adjustable parameters $\hat{\theta}$. For tuning the adjustable parameters, we design the adaptation law. First of all, the dynamics of the disturbance observer error from (7) and (14) is expressed as

$$\dot{\zeta} = -\sigma\zeta + D(z,\tau) - \hat{D}(z,\tau \mid \hat{\theta}) \tag{15}$$

By assumption 2 and the universal approximation capability of the fuzzy system, the unknown disturbance $D(z,\tau)$ can be described by a fuzzy system as follows:

$$\begin{aligned} &D(z,\tau) = \hat{D}(z,\tau \mid \theta^*) + \varepsilon(z,\tau) \\ &|\varepsilon(z,\tau)| \le \bar{\varepsilon} \end{aligned} \tag{16}$$

where the upper bound $\bar{\varepsilon}$ can be reduced arbitrarily by increasing the number of the fuzzy rules. By substituting (16) into (15), the dynamics of the disturbance observation error becomes

$$\dot{\zeta} = -\sigma\zeta + \hat{D}(z,\tau \mid \theta^*) - \hat{D}(z,\tau \mid \hat{\theta}) + \varepsilon(x,u) \tag{17}$$

We rewrite $\hat{D}(z,\tau \mid \theta^*) = \theta^{*T}\xi(x,u)$ and $\hat{D}(z,\tau \mid \hat{\theta}) = \hat{\theta}^T\xi(x,u)$ using representation of fuzzy systems where $\xi(z,\tau)$ is the fuzzy basis function. Define the parameter error $\tilde{\theta} = \theta^* - \hat{\theta}$ and yield

$$\dot{\zeta} = -\sigma\zeta + \tilde{\theta}^T\xi(z,\tau) + \varepsilon(z,\tau) \tag{18}$$

Let the Lyapunov candidate function be given by

$$V = \frac{1}{2}\zeta^2 + \frac{1}{2\gamma}\tilde{\theta}^T\tilde{\theta} \tag{19}$$

Differentiating (19) and substituting (18) into the derivative of (19) yields

$$\begin{aligned} \dot{V} &= \zeta\dot{\zeta} + \frac{1}{\gamma}\tilde{\theta}^T\dot{\tilde{\theta}} \\ &= -\sigma\zeta^2 + \zeta\tilde{\theta}^T\xi(z,\tau) + \zeta\varepsilon(z,\tau) + \frac{1}{\gamma}\tilde{\theta}^T\dot{\tilde{\theta}} \\ &= -\sigma\zeta^2 + \tilde{\theta}^T\left\{\zeta\xi(z,\tau) + \frac{1}{\gamma}\dot{\tilde{\theta}}\right\} + \zeta\varepsilon(z,\tau) \end{aligned} \tag{20}$$

Choosing an adaptation law as

$$\dot{\hat{\theta}} = \gamma\zeta\xi(z,\tau) \tag{21}$$

yields

$$\begin{aligned} \dot{V} &= -\sigma\zeta^2 + \zeta\varepsilon(z,\tau) \\ &= -\frac{\sigma}{2}\zeta^2 + \frac{1}{2\sigma}\varepsilon^2(z,\tau) - \left(\sqrt{\frac{\sigma}{2}}\zeta - \sqrt{\frac{1}{2\sigma}}\varepsilon(z,\tau)\right)^2 \\ &\le -\frac{\sigma}{2}\zeta^2 + \frac{1}{2\sigma}\varepsilon^2(z,\tau) \end{aligned} \tag{22}$$

Thus, $\dot{V}$ is negative for $|\zeta| > \bar{\varepsilon}/\sigma$. Then, under the assumption that $\hat{\theta}$ is bounded, the disturbance observation error is uniformly ultimately bounded.

3. Robust tracking controller based on the proposed FDO

In this section, we discuss the proposed fuzzy-disturbances observer-based robust tracking control for the WMR with skidding and slipping. First of all, we describe the tracking error dynamics of the WMR. Then, if the skidding and slipping are unmeasurable, the estimate of the disturbances can be used for control design using the proposed FDO. The stability of the system is proved in the form of a proposition. The main idea of this section is to develop the proposed FDO and a tuning method so that the developed disturbance observer is guaranteed to monitor and represent the disturbances well. The proposed controller attenuates the disturbance to improve the tracking performance. The configuration of the proposed FDO-based control for the WMR is shown in Figure 1.

A reference trajectory $q_r(t) = [x_r(t) \quad y_r(t) \quad \phi_r(t)]^T$ is represented by

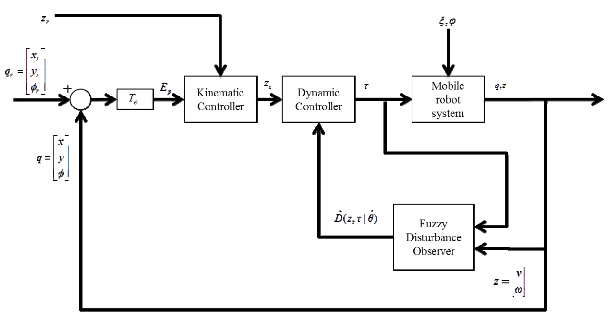


Figure 1. The configuration of the proposed FDO-based control for the WMR

$$\dot{x}_r = v_r \cos\phi_r,\quad \dot{y}_r = v_r \sin\phi_r,\quad \dot{\phi}_r = \omega_r \tag{23}$$

where $z_r = \begin{bmatrix} v_r & \omega_r \end{bmatrix}^T$, v_r is the reference of the forward linear velocity and ω_r is the reference of the angular velocity of mobile robots. A smooth velocity control law $z_c = f_c(E_p, z_r, K_c)$ is determined by $\lim_{t\to\infty} q(t) = q_r(t)$ where E_p, z_r, and $K_c = \begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix}$ are the tracking position error, the reference velocity and the control gain, respectively. We can then find the torque input, which can be computed without measuring velocities such that $z(t) \to z_c(t)$ as $t \to \infty$.

The tracking error is expressed relative to the local coordinate frame fixed on the mobile robot as $E_p = T(q_r - q)$ or

$$E_p = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \phi_r - \phi \end{bmatrix} \tag{24}$$

The error rate becomes

$$\dot{E}_p = \begin{bmatrix} \omega e_1 - v + v_r \cos e_3 \\ \omega e_1 + v_r \sin e_3 \\ \omega_r - \omega \end{bmatrix} \tag{25}$$

An auxiliary velocity control input that achieves tracking for the kinematic model (6) with skidding and slipping, i.e., with which $E_p = 0$ is uniformly asymptotically stable under the assumption $v_r > 0$, is given by

$$z_c = \begin{bmatrix} v_c \\ \omega_c \end{bmatrix} = \begin{bmatrix} v_r \cos e_3 + k_1 e_1 \\ \omega_r + k_2 v_r e_2 + k_3 v_r \sin e_3 \end{bmatrix} \tag{26}$$

where $k_1, k_2, k_3 > 0$ are design parameters. If only the kinematic model of the mobile robot (2) with velocity input (26) is considered, then the kinematic model is asymptotically stable with respect to the reference trajectory [9].

To design the control input and generate the desired velocities z_c, the auxiliary velocity tracking error is defined as

$$e_c = z - z_c \tag{27}$$

Differentiating (27) and using the result in (7), tracking error dynamics of a WMR can be rewritten as

$$\dot{e}_c = A(e_c + z_c) + B\tau + D(z,\tau) - \dot{z}_c \tag{28}$$

where $D(z,\tau)$ is an important nonlinear WMR function for skidding and slipping. To track the reference trajectory, we design a control input τ for $e_c \to 0$, which means that $z(t) \to z_c(t)$ as $t \to \infty$.

Using the fuzzy disturbance observer (14), we can estimate the disturbance (16) and attenuate the disturbance for tracking control. To make the tracking error dynamics stable, the torque term as the input in (28) is chosen as

$$\tau = B^{-1}(\dot{z}_c - Az_c - Ke_c - \hat{D}(z,\tau\,|\,\hat{\theta})) \tag{29}$$

where K is a positive definite matrix. If we choose K properly, we can not only estimate disturbances, but also eliminate them for tracking control of a WMR. It is not necessary to measure the values of ξ, $\dot{\xi}$, φ, and $\dot{\varphi}$. Thus, wheel skidding and slipping information is not required to implement this control law. The proposed controller improves the tracking performance.

We consider a nonholonomic WMR (7) with an unknown disturbance $D(z,\tau)$ caused by skidding and slipping. To construct an FDO $\hat{D}(z,\tau\,|\,\hat{\theta})$ that is guaranteed to monitor an unknown $D(\cdot)$, the adaptation law must be provided for the adjustable parameters $\hat{\Phi}$.

The main objective of this paper is to design the robust tracking controller based on the FDO. In order to achieve this, the Lyapunov function (22) should be modified as the Lyapunov function (31). Then, the adaptation law can be obtained as (30). This is explained in detail in the following theorem.

Theorem 1: Given the nonholonomic system (7), use the proposed control input given by (29). If the adaption law of parameters is given by

$$\dot{\hat{\Phi}} = \Gamma\{R^T(z)\bar{M}^T e_c + \rho R^T(z)\} \tag{30}$$

where Γ is a positive constant design matrix, the tracking error dynamics (19) are asymptotically stable under $|\zeta| > \bar{\varepsilon}/\sigma$.

Proof

Consider the Lyapunov function candidate for tracking control systems of WMR with skidding and slipping

$$V = V_1 + V_2 + V_3 \tag{31}$$

where

$$V_1 = \tfrac{1}{2}(e_1^2 + e_2^2) + \tfrac{1}{k_2}(1 - \cos e_3) \tag{32}$$

$$V_2 = \tfrac{1}{2} e_c^T \bar{M} e_c + \tfrac{1}{2} \tilde{\Phi}^T \Gamma^{-1} \tilde{\Phi} \tag{33}$$

$$V_3 = \tfrac{1}{2} \rho^T \rho + \tfrac{1}{2} \tilde{\Phi}^T \Gamma^{-1} \tilde{\Phi} \tag{34}$$

Obviously, $V > 0$.

Differentiating (31), we obtain

$$\dot{V} = \dot{V}_1 + \dot{V}_2 + \dot{V}_3 \tag{35}$$

Substituting the values from (24) and (25):

$$\dot{V}_1 = -k_1 e_1^2 - \frac{k_3}{k_2} v_r \sin^2 e_3 \tag{36}$$

Since the reference linear velocity $v_r \geq 0$, therefore $\dot{V}_1 \leq 0$. Differentiating (37) and substituting the error dynamics from (19), we obtain

$$\begin{aligned} \dot{V}_2 &= \tfrac{1}{2} e_c^T (\dot{\bar{M}} - 2\bar{V}_m) e_c - e_c^T \bar{M} K e_c \\ &+ e_c^T \bar{M} \tilde{D} + \tilde{\Phi}^T \Gamma^{-1} \dot{\tilde{\Phi}} \end{aligned} \tag{37}$$

Owing to the property $(\dot{\bar{M}} - 2\bar{V}_m)$ is skew symmetric:

$$e_c^T (\dot{\bar{M}} - 2\bar{V}_m) e_c = 0 \tag{38}$$

$$\tilde{D}(z,\tau) = D(z,\tau) - \hat{D}(z,\tau \mid \hat{\theta}) = R(z)\tilde{\Phi} \tag{39}$$

Substituting (38) and (39) into (37),

$$\dot{V}_2 = -e_c^T \bar{M} K e_c + \tilde{\Phi}^T (R^T(z) \bar{M}^T e_c + \Gamma^{-1} \dot{\tilde{\Phi}}) \tag{40}$$

Since $\tilde{\Phi} = \Phi - \hat{\Phi}$, then $\dot{\tilde{\Phi}} = -\dot{\hat{\Phi}}$; therefore, the Fuzzy Logic Systems (FLS) parameter-tuning laws are obtained for tracking control as

$$\dot{\hat{\Phi}} = \Gamma R^T \bar{M}^T e_c \tag{41}$$

When the adaption law is (21), $\dot{V}_3 < 0$ is also proved as in section 2.2 under $|\zeta| > \bar{\varepsilon}/\sigma$. Thus, if we choose the adaption law (30) as the sum of (21) and (41), $\dot{V} \leq 0$. This proves that tracking control for the WMR with skidding and slipping is asymptotically stable.

Note that the previous scheme to solve WMR with skidding and slipping is limited by the assumption that disturbances are bounded and that derivatives of disturbance should converge to zero with time in order to guarantee the stability of the control system [1-6], whereas there is no assumption of disturbance for the stability analysis in the proposed method as shown in Theorem 1.

4. Simulation results

In order to demonstrate the validity of the suggested control scheme, a comparative simulation with the scheme proposed in [1] is performed. An elliptical reference trajectory is used, given by $x_r = 10\cos(t)$, $y_r = 6\sin(t)$, $\phi_r = \tan^{-1}(\dot{y}_r / \dot{x}_r)$. The reference linear and angular velocities (42) are given by

$$\begin{aligned} v_r(t) &= \sqrt{\dot{x}_r(t) + \dot{y}_r(t)} \\ \omega_r(t) &= \frac{\dot{y}_r^2(t)\dot{x}_r^2(t) - \ddot{x}_r^2(t)\dot{y}_r^2(t)}{\dot{x}_r^2(t) + \dot{y}_r^2(t)} \end{aligned} \tag{42}$$

All the parameters can be found in [1]. $\Gamma = diag(30000 \quad 30000)$ and $\sigma = 100$ are chosen by simulations to improve the transient response when we observe the disturbance. These parameters are positive definite matrix and positive constant to stabilize the tracking error and observation error. Fuzzy modelling depends on the membership functions. It is important to design the controller. We select membership functions (43) for the premise parts of the FDO. $\mu_{A_1^i}(v)$ $\mu_{A_d^i}(\omega)$, $i = (1,2,\cdots,5)$ are Gaussian membership functions. These functions are useful to model the unknown systems. These particular parameter values of membership functions depend on the magnitude of disturbances. We choose the particular parameter values of membership functions according to the expected magnitude of disturbances.

$$\begin{aligned} \mu_{A_1^1}(v) &= \exp\left(-(v+15)^2 \big/ 2(5)^2 \right) \\ \mu_{A_1^2}(v) &= \exp\left(-(v+7)^2 \big/ 2(4)^2 \right) \\ \mu_{A_1^3}(v) &= \exp\left(-v^2 \big/ 2(5)^2 \right) \\ \mu_{A_1^4}(v) &= \exp\left(-(v\text{-}8)^2 \big/ 2(4)^2 \right) \\ \mu_{A_1^5}(v) &= \exp\left(-(v\text{-}15)^2 \big/ 2(5)^2 \right) \\ \mu_{A_2^1}(\omega) &= \exp\left(-(\omega+6)^2 \big/ 2(2)^2 \right) \\ \mu_{A_2^2}(\omega) &= \exp\left(-(\omega+3)^2 \big/ 2(1.6)^2 \right) \\ \mu_{A_2^3}(\omega) &= \exp\left(-\omega^2 \big/ 2(2)^2 \right) \\ \mu_{A_2^4}(\omega) &= \exp\left(-(\omega\text{-}3)^2 \big/ 2(1.6)^2 \right) \\ \mu_{A_2^5}(\omega) &= \exp\left(-(\omega-6)^2 \big/ 2(2)^2 \right) \end{aligned} \tag{43}$$

Two comparative simulations are performed, and they can be summarized as the following two cases.

4.1 Case 1: The derivatives of disturbances converge to zero with time

The disturbances used in [1] are given; these disturbances satisfy assumption 1 and their derivatives converge to zero with time, as shown in Figure 2.

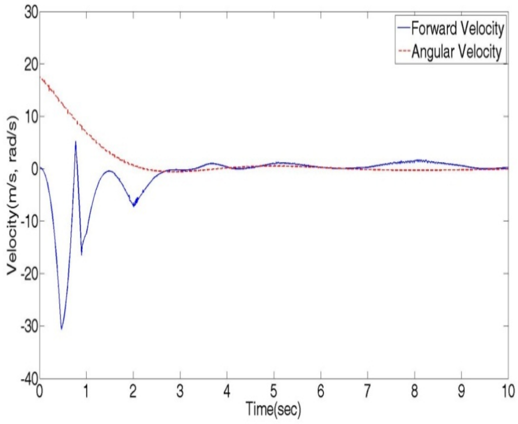


Figure 2. Disturbances of forward and angular velocity caused by skidding and slipping in case 1; In case that the derivatives of disturbances converge to zero as time goes

As Figures 3 and 4 show, both the Generalized Extended State Disturbance Observer (GESDO)-based control method and the proposed control method solve the skidding and slipping problem given in case 1. The performance of GESDO in case 1 has been already proven in [1]. Figure 4 shows the estimation error of disturbances. The proposed controller depends on selecting the membership functions to estimate the disturbances. There seem to be larger forward velocity errors than with the GESDO-based controller from 0.5 to 3 sec. However, the proposed method shows smaller chattering phenomenon than the GESDO method. The GESDO method also depends on the disturbance compensation gain. The tracking control performances are quite similar and it is hard to decide which method is better on this basis. However, it is verified that the proposed control method also achieves the desired goal under the same conditions through this case study. What is noteworthy is that real skidding and slipping do not disappear with time. This means that the derivatives of disturbances never go to zero with time, although they may have small values. Therefore, the disturbances used in case 2 must be considered instead of those used in case 1.

4.2 Case 2: The derivatives of disturbances DO NOT converge to zero with time

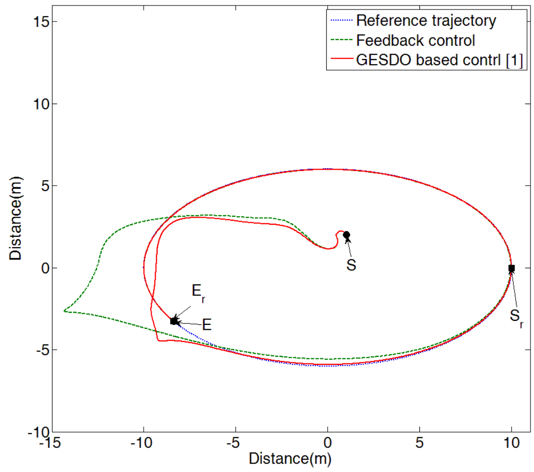
Here, two cases of skidding and slipping are described:

Case 2-1: Skidding and slipping can occur when a WMR turns curves on slippery roads quickly. To implement a simulation of slippery roads, we use the disturbance functions $[\mu \quad \xi_r \quad \xi_l] = [6\cos(t) \quad 6\cos(t) \quad 3]$.

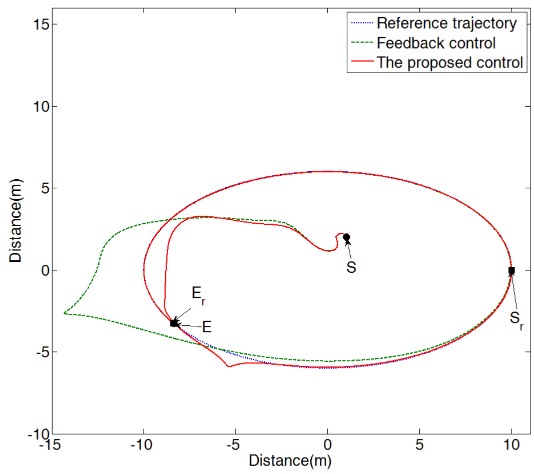
Case 2-2: Periodic skidding and slipping can occur on icy roads. We select the disturbance functions $[\mu \quad \xi_r \quad \xi_l] = [4\cos(5t) \quad 4\sin(5t) \quad 6\cos(5t)]$.

The above two cases satisfy assumption 1, and these derivatives of disturbances do not converge to zero with time, as shown in Figures 5 and 9.

The objective of this study is to verify that the proposed method overcomes more varied disturbances and is more useful in practice than our previous work. The forward and angular velocity disturbances caused by skidding and slipping given in case 2 are shown in Figures 5 and 9. As already mentioned, their derivatives do not go to zero, which means disturbances exist while WMRs follow the desired trajectory.



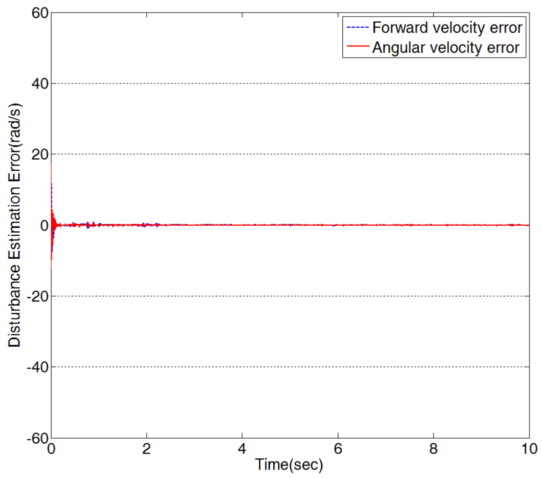
(a) Result of GESDO-based control method



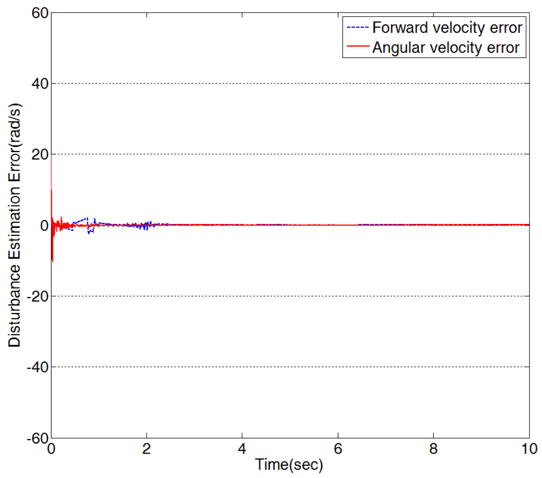
(b) Results of the proposed control method

Figure 3. Trajectory-tracking control performance of the WMR in case 1 (S: Start-point of a WMR, E: End-point of a WMR, S_r: Start-point of reference, E_r: End-point of reference)

In cases where the derivatives of disturbances DO NOT converge to zero with time, Figures 6 and 10 present the trajectory-tracking performance of both methods according to each disturbance. Since the GESDO-based control method assumes that the derivatives of disturbance should converge to zero with time, and the guarantee of its stability relies on this, the tracking result diverges. On the other hand, the proposed controller overcomes the same problem because it adopts a fuzzy model, which is a universal approximation model for designing a disturbance observer. This means no assumption about disturbances is necessary. Figures 7 and 11 show disturbance estimation performance and describe the estimation error. That of the proposed controller goes to zero or is bounded in a small region, whereas the estimation error of the GESDO-based controller does not; the GESDO-based controller thus fails to achieve the control goal. Figures 8 and 12 show the trajectory-tracking errors of the WMR. The proposed control method is less dependent on disturbance type than the method presented in [1].



(a) Results of GESDO-based control method



(b) Results of the proposed control method

Figure 4. Disturbance estimation errors in case 1

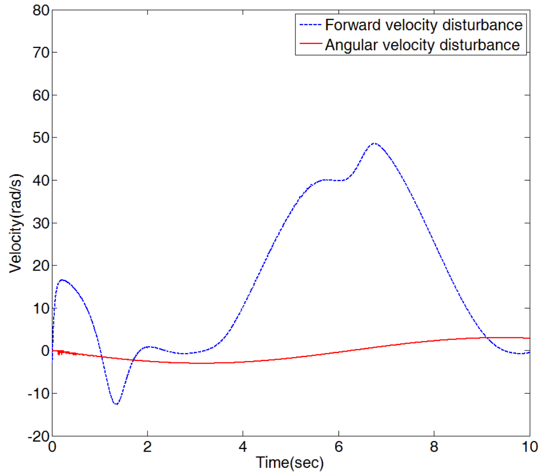
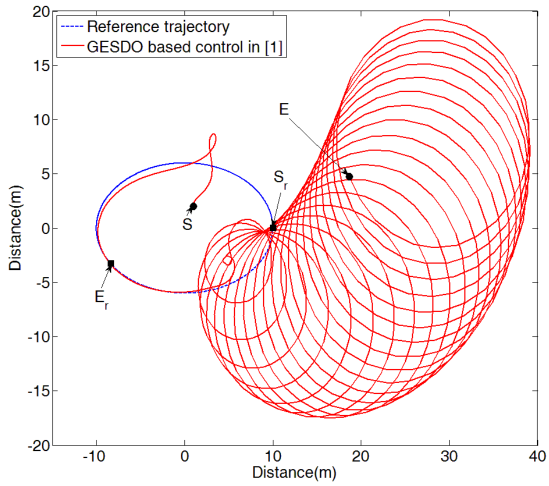
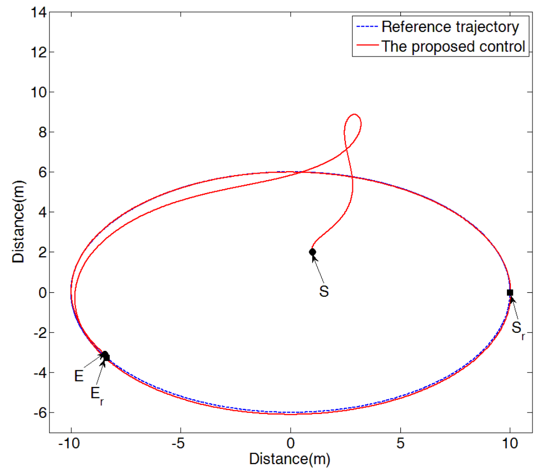


Figure 5. Disturbances of forward and angular velocity caused by skidding and slipping in case 2-1

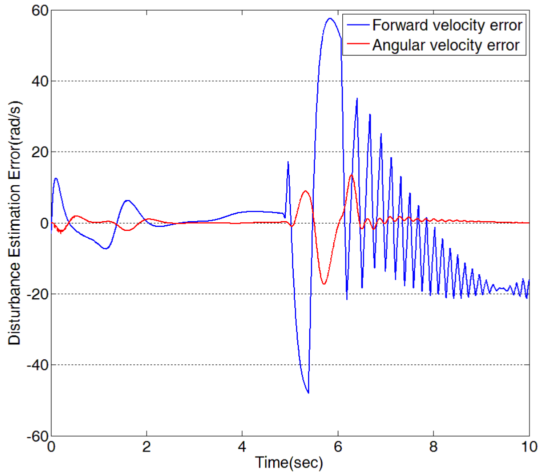


(a) Results of GESDO-based control method

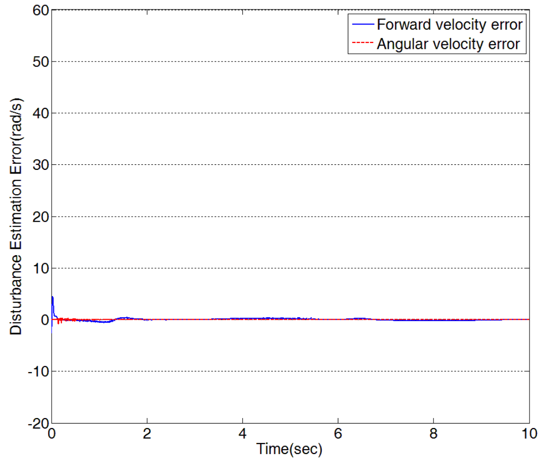


(b) Results of the proposed control method

Figure 6. Trajectory-tracking control performance of the WMR in case 2-1 (S: Start-point of a WMR, E: End-point of a WMR, S_r: Start-point of reference, E_r: End-point of reference)

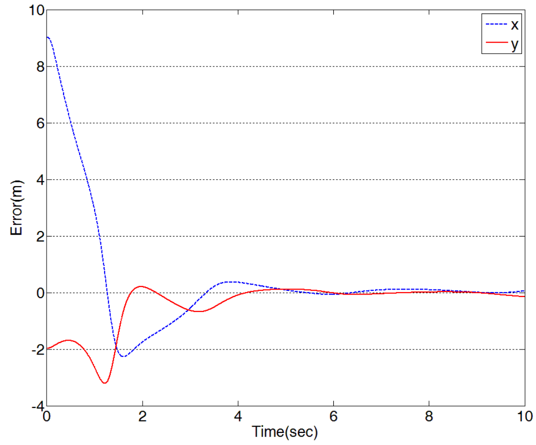


(a) Results of GESDO-based control method



(b) Results of the proposed control method

Figure 7. Disturbances estimation errors in case 2-1



(b) The result of the proposed control method

Figure 8. Trajectory-tracking error of the WMR in case 2-1

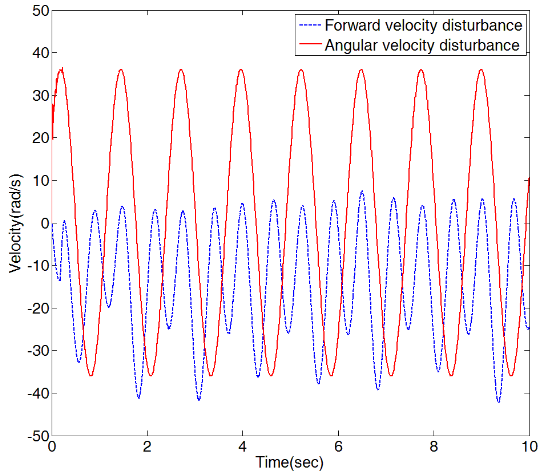
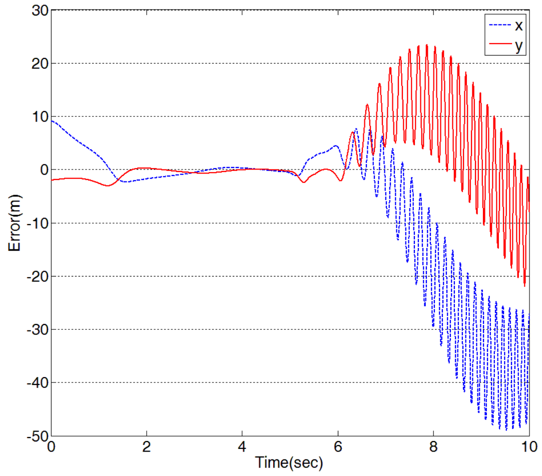
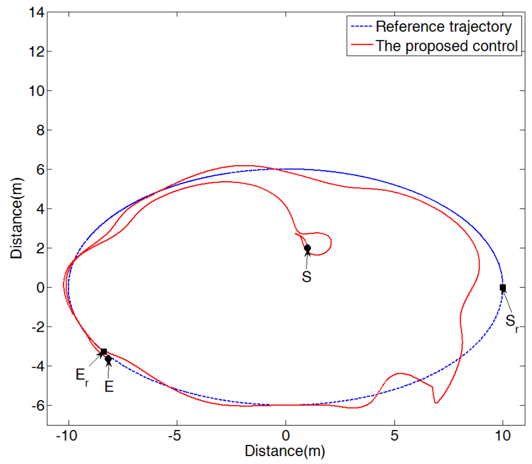


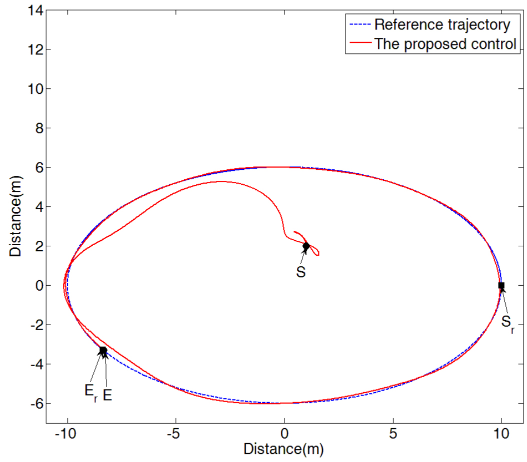
Figure 9. Disturbances of forward and angular velocity caused by skidding and slipping in case 2-2



(a) Results of GESDO-based control method

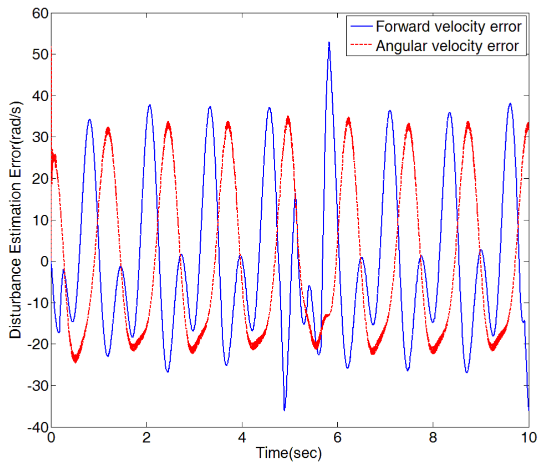


(a) Results of GESDO-based control method

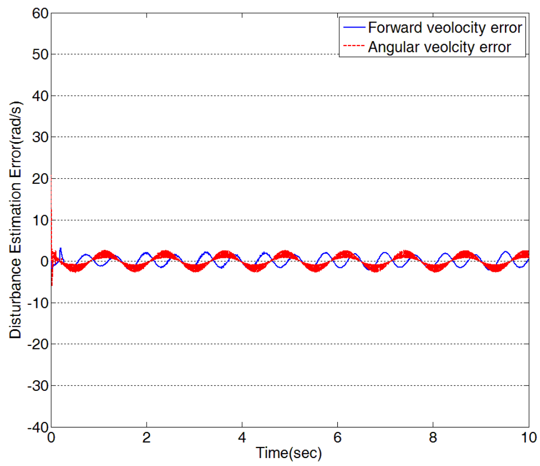


(b) The result of the proposed control method

Figure 10. Trajectory-tracking control performance of the WMR in case 2-2 (S: Start-point of a WMR, E: End-point of a WMR, S_r: Start-point of reference, E_r: End point of reference)

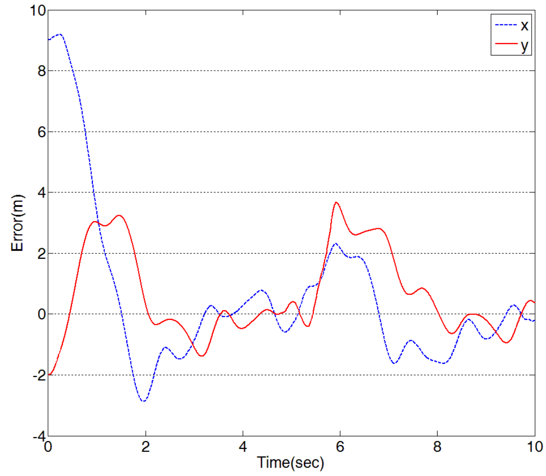


(a) Result of GESDO-based control method

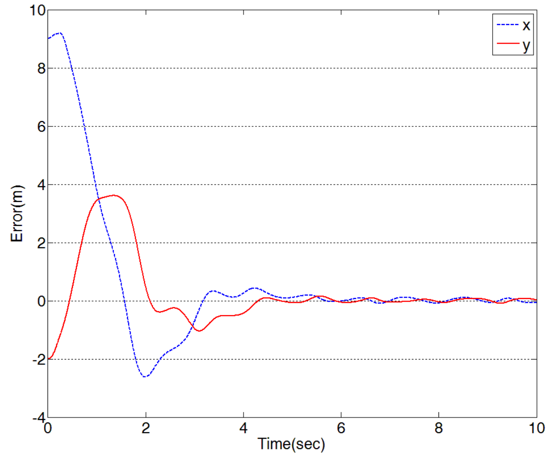


(b) Results of the proposed control method

Figure 11. Disturbances estimation errors in case 2-2



(a) Result of GESDO-based control method



(b) Results of the proposed control method

Figure 12. Trajectory-tracking error of the WMR in case 2-2

5. Conclusion

In this paper, a robust tracking controller based on the FDO for a WMR with unknown skidding and slipping is proposed as an expansion of our previous work. A universal approximation, fuzzy modelling is adopted to design a disturbance observer. It frees the proposed method from any disturbance assumption to analyse the stability and design a controller with wide applicability. Through two case studies, it was shown that real skidding and slipping do not disappear with time, which means that the derivatives of disturbances never go to zero with time. In addition, it was verified that the proposed control method achieves the desired goal under such conditions. Consequently, the results verified that the proposed controller provides robustness against unknown skidding and slipping without assuming them, and shows significant tracking performance.

6. Acknowledgements

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7. References

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